UDC 532.7:534.1 DOI: https://doi.org/10.15407/geotm2022.163.174

OSCILLATORY LIQUID MOTION IN CAPILLARIES, THE GEOMETRY OF WHICH CHANGES WEAKLY

[1]Yelisieiev V.I., [1]Lutsenko V.I., [2]Berkout V.D.

[1]Institute of Geotechnical Mechanics named by N. Poljakov of National Academy of Sciences of Ukraine; [2]MassTech Inc.

Abstract. In this paper, on the basis of previously performed studies, the oscillatory motions of a liquid in a narrow channel, the width of which varies slightly, are considered. The solution of the problem was carried out on the basis of the boundary layer model for the case of a flat arrangement, which to some extent simplified the analytical description compared to the axisymmetric problem and at the same time did not qualitatively change the physical features of the investigated process. Asymptotic methods with a small parameter were used to obtain solutions. Three different variants of the task are considered, which differ from each other in the patterns of changes in the width of the channel along the longitudinal axis. As a result of the solution, it is shown that when the width of the channel changes monotonically, the ratio of pressure gradients also changes monotonically. When the width of the channel is the same at the ends of the analyzed section, the ratio of pressure gradients at the ends of the channel is also equal. With a sinusoidal change in the channel width, changes in pressure gradients also have a sinusoidal character. The main result of the work is the conclusion that when the width of the flat capillary changes, the phase shift of the pressure fluctuation relative to itself changes, at the same time the flow rate fluctuation relative to the pressure also changes, but the sum of these shifts remains a constant value. In known solutions for a constant channel diameter, the harmonic oscillations of flow and pressure also have a phase shift relative to each other that depends on the channel diameter but does not vary along the channel. It is also shown that with further approximations, components containing harmonics with a doubled frequency and a component that does not depend on time appear in the solution, i.e. with an oscillating flow in a capillary of non-constant width, a time-independent flow is formed in certain zones. The structure of such an internal flow, as follows from the solution, depends on the change in the diameter of the capillary and on the specified oscillation.

Keywords: capillary, liquid, diffusion, mass exchange, oscillations.

1. Introduction

The study of oscillatory motion is of great interest, both in itself and for a wide range of applications, and in particular, for heat and mass transfer processes. The study of the oscillatory nature of the flow in capillary systems is related to many natural and technological processes. In the mining industry, this is directly related to the technologies for preparing mineral raw materials for further processing and the technologies for using secondary resources. In addition, researchers are currently paying a lot of attention to the study of processes in biological systems, in particular, to the study of capillary processes in plants and the circulatory systems of animals and humans.

In most cases, for the detailed study of non-stationary flows in porous media, the tasks are reduced to analysis of capillary flows in individual tubes. In this case, the topologically complex pore system is reduced to some simple connection of capillaries [1]. Among various tasks related to the dynamic behavior of the system and heat and mass exchange processes, tasks with non-stationary and, in particular, pulsating fluid flows are of considerable interest [2], [3]. The range of applications of such tasks is quite wide, from geotechnical [4], [5] and agrotechnical problems [6] to biological systems [7].

In [8], [9], [10] classical formulations of such tasks are given. With the development of numerical methods and computer technology, the range of research topics has expanded significantly, however, previously developed analytical and approximate methods have not lost their importance and can be used in modern settings for understanding of the physical features of motion.

As follows from the known solutions at a constant diameter, harmonic oscillations of flow and pressure have a phase shift relative to each other that does not change along the channel [3]. If the diameter of the capillary has any changes, then this will affect the nature of the oscillations in different parts of the channel. Capillaries of living systems under the influence of mechanical forces [11], and those in quasi-stationary states, for example, in tree trunks [12], have a changing channel geometry. Subsequently, the effect of variable channel geometry on the nature of oscillation in narrow channels is of interest.

In this work, the oscillatory movements of liquid in capillaries with a channel width that varies slightly along the longitudinal axis are considered. The solution of the problem is carried out for the case of a flat arrangement, since this to a certain extent simplifies its analytical description in comparison with an axisymmetric problem, and on the other hand, it does not qualitatively change the physical features of the process being studied.

2. Methods

The basic equations of unsteady motion in a narrow channel with a small width change according to the boundary layer model [8] are:

$$\frac{\partial u}{\partial t}+\frac{\partial\left(u^2\right)}{\partial x}+\frac{\partial(vu)}{\partial y}=-\frac{\partial p}{\rho\partial x}+\frac{\mu}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right), \quad (1)$$

$$\frac{\partial(u)}{\partial x}+\frac{\partial(v)}{\partial y}=0, \quad (2)$$

where t is time, x is the longitudinal coordinate along the axis of the channel, y is the transverse coordinate, u and v are the longitudinal and transverse velocities, ρ is the density, p is the pressure, and μ is the dynamic coefficient of viscosity.

Next, we assume that the width of the channel changes slowly along the longitudinal coordinate, i.e. $h = h_0(1 + \delta)$, where h_0 is the characteristic width of the channel, and δ is a function of the dimensionless quantity $\zeta = \beta x/h_0$, $\beta << 1$ is a parameter. This condition allows the use of asymptotic methods [13].

At a constant width of the channel, the functions u, v, and $\frac{\partial p}{\rho\partial x}$ do not depend on x. With a weakly changing width, these values will depend on ζ, therefore, after introducing another dimensionless value $n = y/h$, we can rewrite equations (1), (2) in the following form:

$$h_0^2(1+\delta)^2\frac{\partial u}{\partial t}+h_0(1+\delta)^2\beta\left(\frac{\partial\left(u^2\right)}{\partial\zeta}-n\frac{\delta'}{(1+\delta)}\frac{\partial\left(u^2\right)}{\partial n}\right)+h_0(1+\delta)\frac{\partial(vu)}{\partial y}=$$

$$= -h_0^{\ 2}(1+\delta)^2 P + \frac{\mu}{\rho}\frac{\partial}{\partial n}\left(\frac{\partial u}{\partial n}\right); \tag{3}$$

$$(1+\delta)\beta\left(\frac{\partial(u)}{\partial\zeta} - n\frac{\delta^{/}}{(1+\delta)}\frac{\partial(u)}{\partial n}\right) + \frac{\partial(v)}{\partial n} = 0\,. \tag{4}$$

In these equations $\delta^{/} = d\,\delta/d\,x$.

The condition of the smallness of the β parameter makes it possible to construct the asymptotic solution of equations (3), (4). For oscillatory motion, the solution of these equations can be written in the form of expansions by powers of the small parameter β:

$$u = u_0(n,t) + \beta u_1(\zeta,n,t) + \ldots\,, \quad u_0 = Us_0\ Sin(2\pi ft) + Uc_0\ Cos(2\pi ft),$$

$$v = \beta v_1(\zeta,n,t) + \beta^2 v_2 + \ldots, \qquad v_1 = [Vs_1\ Sin(2\pi ft) + Vc_1\ Cos(2\pi ft)], \tag{5}$$

$$P = P_0(\zeta,t) + \beta P_2(\zeta,t) + \ldots\,, \qquad P_0 = Ps_0\ Sin(2\pi ft) + Pc_0\ Cos(2\pi ft).$$

Here f is the oscillation frequency; values Us_0, Uc_0, Vs_1, Vc_1, Ps_0, Pc_0 are functions of n determined during the solution.

After substituting these expressions in the zero approximation, we get the following equations:

$$-\chi^2 Uc_0 = -\frac{\chi^2}{2\pi f}Ps_0 + \frac{\partial^2 Us_0}{\partial n^2}, \tag{6}$$

$$\chi^2 Us_0 = -\frac{\chi^2}{2\pi f}Pc_0 + \frac{\partial^2 Uc_0}{\partial n^2}, \tag{7}$$

where $\chi^2 = 2\pi f h_0^{\ 2}(1+\delta)^2(\rho/\mu)$. After substitution $Uc_0 = \frac{1}{2\pi f}Ps_0 + Unc_0$ and $Us_0 = -\frac{1}{2\pi f}Pc_0 + Uns_0$, we get an equation in the form

$$\frac{\partial^4 Unc_0}{\partial n^2} + \chi^4 Unc_0 = 0\,, \tag{8}$$

whose solution is an expression

$$
\begin{aligned}
Unc_0 &= A\, exp\, \lambda n\, Cos\, \lambda n + B\, exp\, \lambda n\, Sin\, \lambda n + \\
&+ C\, exp(-\lambda n) Cos\, \lambda n + D\, exp(-\lambda n) Sin\, \lambda n
\end{aligned}
\qquad (9)
$$

and then

$$
\begin{aligned}
Uns_0 &= -A\, exp\, \lambda n\, Sin\, \lambda n + B\, exp\, \lambda n\, Cos\, \lambda n + \\
&+ exp(-\lambda n) Sin\, \lambda n - D\, exp(-\lambda n) Cos\, \lambda n
\end{aligned}
, \qquad (10)
$$

where $\lambda^2 = 0.5\chi^2$, A, B, C, D – coefficients.

From symmetry conditions on the axis of the channel and equality of zero longitudinal velocities on the walls, we find the following relations $B + D = 0$, $A - C = 0$ and

$$
\begin{aligned}
&\left\{[exp\, \lambda + exp(-\lambda)]^2 Cos^2\, \lambda + [exp\, \lambda - exp(-\lambda)]^2 Sin^2\, \lambda\right\} \cdot B = \\
&= \frac{1}{2\pi f}\left\{[exp\, \lambda + exp(-\lambda)] Cos\, \lambda \cdot Pc_0 - [exp\, \lambda - exp(-\lambda)] Sin\, \lambda \cdot Ps_0\right\},
\end{aligned}
$$

$$
\begin{aligned}
&\left\{[exp\, \lambda + exp(-\lambda)]^2 Cos^2\, \lambda + [exp\, \lambda - exp(-\lambda)]^2 Sin^2\, \lambda\right\} \cdot A = \\
&= -\frac{1}{2\pi f}\left\{[exp\, \lambda + exp(-\lambda)] Cos\, \lambda \cdot Ps_0 + [exp\, \lambda - exp(-\lambda)] Sin\lambda \cdot Pc_0\right\}.
\end{aligned}
$$

Next, by determining the volume flow in the channel in the form of

$$
Qc_0 = h\int_0^1 Us_0\, dn = h_0\, \frac{(1+\delta)}{2\pi f} Kcs \cdot Ps_0 + h_0\, \frac{(1+\delta)}{2\pi f} Kcc \cdot Pc_0, \qquad (11)
$$

$$
Qs_0 = h\int_0^1 Uc_0\, dn = h_0\, \frac{(1+\delta)}{2\pi f} Kss \cdot Ps_0 - h_0\, \frac{(1+\delta)}{2\pi f} Ksc \cdot Pc_0, \qquad (12)
$$

where

$$
Kcs = \left\{
\begin{aligned}
&1 - \frac{1}{2\lambda\, Det}\{exp\, \lambda[Sin\, \lambda + Cos\, \lambda] + exp(-\lambda)[Sin\, \lambda - Cos\, \lambda]\}[exp\, \lambda + exp(-\lambda)] Cos\, \lambda - \\
&- \frac{1}{2\lambda\, Det}\{exp\, \lambda[Sin\, \lambda - Cos\, \lambda] - exp(-\lambda)[Sin\, \lambda + Cos\, \lambda] + 2\}[exp\, \lambda - exp(-\lambda)] Sin\, \lambda
\end{aligned}
\right\},
$$

$$
Kcc = \frac{1}{2\lambda\, Det}\left\{
\begin{aligned}
&\{exp\, \lambda[Sin\, \lambda - Cos\, \lambda] - exp(-\lambda[Sin\, \lambda + Cos\lambda]) + 2\} \cdot [exp\, \lambda + exp(-\lambda)] Cos\, \lambda - \\
&- \{exp\, \lambda[Sin\, \lambda + Cos\, \lambda] + exp(-\lambda)[Sin\, \lambda - Cos\, \lambda]\} \cdot [exp\, \lambda - exp(-\lambda)] Sin\, \lambda
\end{aligned}
\right\},
$$

$$Ksc = \begin{Bmatrix} 1-\dfrac{1}{2\lambda\, Det}\{exp\,\lambda[Sin\,\lambda - Cos\,\lambda] - exp(-\lambda)[Sin\,\lambda + Cos\,\lambda] + 2\}[exp\,\lambda - exp(-\lambda)]Sin\,\lambda - \\ -\dfrac{1}{2\lambda\, Det}\{exp\,\lambda[Sin\,\lambda + Cos\,\lambda] + exp(-\lambda)[Sin\,\lambda - Cos\,\lambda]\}[exp\,\lambda + exp(-\lambda)]Cos\,\lambda \end{Bmatrix},$$

$$Kss = \frac{1}{2\lambda\, Det}\begin{Bmatrix} \{exp\,\lambda[Sin\,\lambda - Cos\,\lambda] - exp(-\lambda)[Sin\,\lambda + Cos\,\lambda] + 2\}[exp\,\lambda + exp(-\lambda)]Cos\,\lambda - \\ -\{exp\,\lambda[Sin\,\lambda + Cos\,\lambda] + exp(-\lambda)[Sin\,\lambda - Cos\,\lambda]\}[exp\,\lambda - exp(-\lambda)]Sin\,\lambda \end{Bmatrix},$$

$$Det = \left\{[exp\,\lambda + exp(-\lambda)]^2 Cos^2\lambda + [exp\,\lambda - exp(-\lambda)]^2 Sin^2\lambda\right\}$$

we will find the relations between pressure gradients and flow rates.

The expressions for *Kss*, *Kcc*, *Ksc*, *Kcs* show that *Kss* = *Kcc* and *Ksc* = *Kcs*. Let's rewrite relations (11), (12) in this form:

$$(Kss \cdot Kcc + Ksc \cdot Kcs)Ps_0 = \frac{2\pi f}{h_0(1+\delta)}(Ksc \cdot Qc_0 + Kcc \cdot Qs_0), \qquad (13)$$

$$(Kss \cdot Kcc + Ksc \cdot Kcs)Pc_0 = \frac{2\pi f}{h_0(1+\delta)}(Kss \cdot Qc_0 - Kcs \cdot Qs_0). \qquad (14)$$

It should be emphasized that the volume flow rates Qs_0 and Qc_0 are unchanged, i.e. are independent on the longitudinal coordinate, which is a consequence of the equality of the transverse velocities on the axis and walls of the channel. However, the coefficients *Kss*, *Ksc*, *Kcc*, *Kcs* slowly change along the longitudinal axis. This indicates that the pressure gradients Ps_0 and Pc_0 should also vary slightly along the channel. An important conclusion follows from this that in capillaries with a non-constant diameter, the pressure gradients along the channel change, i.e. if a certain harmonic pressure oscillation law is set at one end of the capillary, then at the other end this oscillation can be shifted in phase.

3. Results and discussion

Let's give some examples. Let's consider three options: the first – $\delta_1 = a_1\, exp\,\zeta / (1 + exp\,\zeta)$; second – $\delta_2 = a_2\zeta^2 / (1+\zeta^2)$ and the third $\delta_3 = a_3 Sin\zeta$.

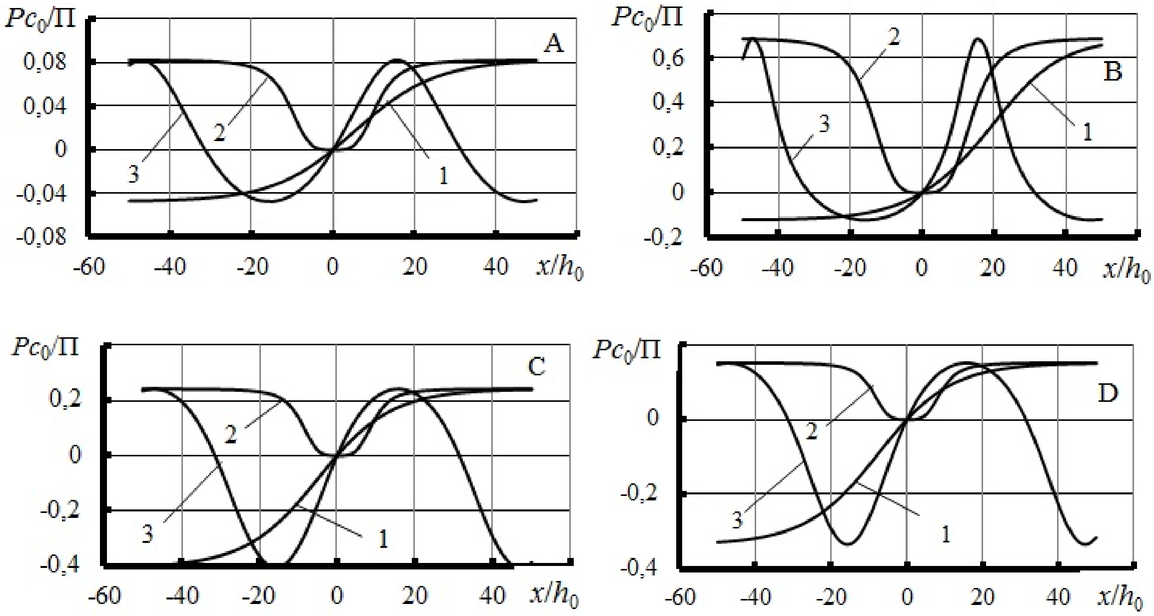
We will conditionally set the total flow, which is determined by the expression

$$G = \sqrt{Qs_0{}^2 + Qc_0{}^2} = h_0\frac{(1+\delta)}{2\pi f}\sqrt{Kss^2 + Ksc^2}\,\Pi, \qquad (15)$$

where $\Pi = \sqrt{Ps_0{}^2 + Pc_0{}^2}$.

From condition (15), we determine the change in value along the axis of the channel. Then, putting $Pc_0 = 0$ at some point, for example, at the point $x = 0$ (this is a formal choice of the initial point on the time axis) from equations (11), (12), we determine the values of Qs_0 and Qc_0 and, keeping them constant, we find from (13), (14) values of Pc_0 and Ps_0 along the entire length of the channel.

The figure below, as an example, shows the $Pc_0/П$ curves along the channel axis for the above options. In order to control the accuracy of the calculations, we choose h_0 and a in such a way that we have the same width of the channel for all options at the control points.



A – f = 1Гц, $h_{01} = 0.0001$, $a_1 = 1$; $h_{02} = 0.00015$, $a_2 = 1/3$; $h_{03} = 0.00015$, $a_3 = 1/3$
B – f = 2 Гц, $h_{01} = 0.0001$, $a_1 = 1$; $h_{02} = 0.00015$, $a_2 = 1/3$; $h_{03} = 0.00015$, $a_3 = 1/3$
C – f = 1Гц, $h_{01} = 0.0005$, $a_1 = 1$; $h_{02} = 0.00075$, $a_2 = 1/3$; $h_{03} = 0.00075$, $a_3 = 1/3$.
D – f = 2 Гц, $h_{01} = 0.0005$, $a_1 = 1$; $h_{02} = 0.00075$, $a_2 = 1/3$; $h_{03} = 0.00075$, $a_3 = 1/3$.

Figure – Change in $Pc_0/П$ curves along the channel axis

In the first series of calculations, it is accepted that $h_{01} = 0.0001$, $a_1 = 1$; $h_{02} = 0.00015$, $a_2 = 1/3$; $h_{03} = 0.00015$, $a_3 = 1/3$, in the second series: $h_{01} = 0.0005$, $a_1 = 1$; $h_{02} = 0.00075$, $\alpha_2 = 1/3$; $h_{03} = 0.00075$, $a_3 = 1/3$. These values give the condition of equality of h at the point $\zeta = 0$ for all three channel configurations (in the first version, $h = 0.00015$; in the second – $h = 0.00075$). In addition, at large positive values of ζ, the values $h = 0.0002$ (in the first series) and $h = 0.001$ (in the second series) are reached for a sinusoidal change - these are the maximum points. Now, with these values, the ratio $Pc_0/П$ should also be equal to what we observe in the figure in all variants. For curves 1 in the left parts of the figure (A, B) $h = 0.0001$ and $h = 0.0005$ (C, D), the minimum values of curves 3 correspond to the same values, so the $Pc_0/П$ values also coincide.

It also follows from the figure that in the first case, when the width of the channel changes monotonically, the ratio of pressure gradients also changes monotonically. In the second case, we have the same width of the channel at the ends of the analyzed section, which also affects the gradients, i.e. they are equal at the ends of the channel. In the third variant, curve 3 also tracks the sinusoidal change in channel width. Thus, when the diameter of the capillary changes, the pressure fluctuation curves at each point have a displacement relative to each other. In our problem, this can be written as:

$$P_0 = \Pi \cdot Sin[2\pi ft + \alpha(\zeta)] = Ps_0 \cdot Sin(2\pi ft) + Pc_0 \cdot Cos(2\pi ft),$$

$$Ps_0 = \Pi \cdot Cos\,\alpha\,,$$

$$Pc_0 = \Pi \cdot Sin\,\alpha\,.$$

For flow:

$$Q = G \cdot Sin[2\pi ft + \alpha(\zeta) + \theta(\zeta)] = Qs_0 \cdot Sin(2\pi ft) + Qc_0 \cdot Cos(2\pi ft),$$

$$Qs_0 = G \cdot Cos(\alpha + \theta),\; Qc_0 = G \cdot Sin(\alpha + \theta),$$

with $tg\,\theta = \dfrac{Qc_0 / Qs_0 - tg\,\alpha}{1 + Qc_0 / Qs_0 \cdot tg\,\alpha}$.

Here, α is the displacement in pressure fluctuations, which varies with the width of the channel, θ is the displacement in volume flow fluctuations relative to pressure fluctuations. Now, a simple conclusion follows from this. Since when the channel width changes, Qs_0 and Qc_0 remain constant, so tg(α+θ) remains constant, that is, α+θ is also a constant value.

Let us now schematically show another interesting effect. If we take into account the following approximation with respect to β including nonlinear terms, the equations (3), (4) can be written in the following form:

$$h_0{}^2(1+\delta)^2 \frac{\partial u_1}{\partial t} - \frac{\mu}{\rho}\frac{\partial}{\partial n}\left(\frac{\partial u_1}{\partial n}\right) + h_0{}^2(1+\delta)^2 P_1 = - \\ - h_0(1+\delta)^2\left(\frac{\partial\left(u_0{}^2\right)}{\partial \varsigma} - n\frac{\delta^{/}}{(1+\delta)}\frac{\partial\left(u_0{}^2\right)}{\partial n}\right) - h_0(1+\delta)\frac{\partial(v_1 u_0)}{\partial y}, \tag{16}$$

$$\frac{\partial(v_1)}{\partial n} = -(1+\delta)\left(\frac{\partial(u_0)}{\partial \zeta} - n\frac{\delta^{/}}{(1+\delta)}\frac{\partial(u_0)}{\partial n}\right). \tag{17}$$

If we substitute the found terms of zeroth-order to equations (16), (17), then in the right-hand parts we get terms in the form of $Sin^2(2\pi ft)R_1(\zeta, n)$, $Sin(2\pi ft)Cos(2\pi ft)R_2(\zeta, n)$, $Cos^2(2\pi ft)R_3(\zeta, n)$, which, taking into account the properties of harmonic functions, can be transformed as $0.5(R_1(\zeta, n) + R_3(\zeta, n))$, $0.5Cos(4\pi ft)(R_3(\zeta, n) - R_1(\zeta, n))$, $0.5Sin(4\pi ft)R_2(\zeta, n)$. In the first approximation the solution must be sought in the form of $u_1 = U_1(\zeta, n) + Us_1 Sin(4\pi ft) + Uc_1 Cos(4\pi ft)$, and it follows from this that in the solution there are: firstly, terms containing harmonics with a doubled frequency and, secondly, a term that depends on time, i.e., during an oscillating flow in a capillary of variable diameter, a time-independent flow is organized, it exists in those areas where $\delta^{/} \neq 0$. The structure of such an internal flow, as follows from the solution, depends on the change in the diameter of the capillary and on the specified oscillation. The appearance of such effects during wave oscillations of a liquid is quite well known and is determined by means of wave analysis of solutions [14].

4. Conclusions

In this work, the previously discussed problem of oscillations in capillary channels is extended to the case when the width of the channel changes along the longitudinal axis. An asymptotic solution is built for the cases when these changes are smooth and insignificant compared to the length of the considered channel segment. It is shown that a change in channel width leads to a phase shift of pressure fluctuations, which depends on the local value of this parameter, while the sum of phase shifts characterizing the curves of pressure and flow fluctuations remains constant. In addition, the analysis of the solution with taking into account the following approximation shows that an internal flow independent of time is formed in certain zones of a narrow channel of variable cross-section during an oscillating flow of liquid. The structure of this flow depends on the law of change of the channel width and on the specified oscillation parameters.

REFERENCES

1. Kheifets, L.I. and Neimark, A.V. (1982), *Mnogofaznyye protsessy v poristykh sredakh* [Multiphase processes in porous media], Khimiya, Moscow, Russia.

2. Lutsenko, V., Yelisieiev, V. and Sovit, Yu. (2022), "Influence of low-frequency vibrations on mass transfer in pores", *III International Conference "Essays of Mining Science and Practice"*, Dnipro, Ukraine, 2021, https://doi.org/10.1088/1755-1315/970/1/012020

3. Yelisieiev, V.I., Lutsenko, V.I. and Sovit, Yu.P (2022), "Diffusion mass transfer at low-frequency liquid oscillations in capillaries", *Geotekhnicheskaya Mekhanika* [Geo-Technical Mechanics], no. 161, pp. 43–50.

4. Xin, Huang, Tong, Wang, Yanbin, Luo and Jiang, Guo (2023), "Study on the Influence of Water Content on Mechanical Properties and Acoustic Emission Characteristics jf Sandstone: Case Study from China Based on a Standstonefrom Nanyang Area", *Sustainabilite*, vol. 15, issue 1, 552, https://doi.org/10.3390/su15010552

5. Bulat, A.F., Nadutyy, V.P., Yeliseyev, V.I. and Lutsenko, V.I. (2017), *Kapillyarnyye effekty v dinamicheskikh protsessakh obezvozhivaniya izmelchennoy gornoy massy* [Capillary effects in dynamic processes of dehydration of ground rock mass], Porohy, Dnipro, Ukraine.

6. Ryzhova, N.V. and Shutov, V.V. (2009), *Fizika drevesiny* [Wood physics], study guide, KSTU, Kostroma, Russia.

7. Koshelev, V.B., Mukhin, S.I., Sosnin, N.V. and Favorsky, A.P. (2010), *Matematicheskiye modeli kvaziodnomernoy gemodinamiki* [Mathematical models of quasi-one-dimensional hemodynamics], toolkit, MAKS Press, Moscow, Russia.

8. Loytsyanskiy, L.G. (2003), *Mekhanika zhidkosti i gaza* [Mechanics of liquid and gas], Drofa, Moscow, Russia.

9. Galitseyskiy, B. M., Ryzhov, Yu. A. and Yakush, E. V. (1977), *Teplovyye i gidrodinamicheskiye protsessy v koleblyushchikhsya potokakh* [Thermal and hydrodynamic processes in oscillating flows], Mechanical Engineering, Moscow, Russia.

10. Slezkin, N.I. (1955), *Dinamika vyazkoy neszhimayemoy zhidkosti*, [Dynamics of a viscous incompressible fluid], Gostekhizdat, Moscow, Russia.

11. Gamilov, T.M (2017), “Mathematical modeling of blood flow during mechanical effects on blood vessels”, Ph.D. Thesis, Mathematical modeling, numerical methods, software packages, MFTI, Moscow, Russia.

12. Vernigorov, Yu.M. and Kipnis, I.A. (2012), “Capillary model of tree trunks”, *Vestnik of Don State Technical University*, vol. 12, no. 6(67), pp. 24–40.

13. Koul, Dzh. (1972), *Metody vozmushcheniy v prikladnoy matematike* [Perturbation Methods in Applied Mathematics], Mir, Moscow, Russia.

14. Sretensky, L.N. (1977), *Teoriya volnovykh dvizheniy zhidkosti* [Theory of wave motions of a fluid], Nauka, Moscow, Russia.

About authors

Yelisieiev Volodymyr Ivanovych, Candidate of Physics and Mathematics Sciences (Ph.D), Senior Researcher, Senior Researcher in Department of Mine Energy Complexes, Institute of Geotechnical Mechanics named by N. Poljakov of National Academy of Sciences of Ukraine, (IGTM NAS of Ukraine), Dnipro, Ukraine, VIYelisieiev@nas.gov.ua

Lutsenko Vasyl Ivanovych, Candidate of Technical Sciences (Ph.D), Senior Researcher, Senior Researcher in Department of Mine Energy Complexes, Institute of Geotechnical Mechanics named by N. Poljakov of National Academy of Sciences of Ukraine, (IGTM NAS of Ukraine), Dnipro, Ukraine, VILutsenko@nas.gov.ua

Berkout Vadym Dmytrovych, Candidate of Physics and Mathematics Sciences (Ph.D), Senior Researcher, Senior Researcher, MassTech Inc. (MTI), Columbia, Maryland USA, vberkout@yahoo.com

ВІБРАЦІЙНИЙ РУХ РІДИНИ У КАПІЛЯРАХ, ГЕОМЕТРІЯ ЯКИХ СЛАБО ЗМІНЮЄТЬСЯ

Єлісєєв В.І., Луценко В.І., Беркут В.Д.

Анотація. У цій роботі на основі виконаних раніше досліджень розглянуті коливальні рухи рідини у вузькому каналі, ширина яких слабо змінюється. Розв'язання задачі проведено на основі моделі прикордонного шару для випадку плоскої постановки, що певною мірою спростило аналітичний опис порівняно з осесиметричним завданням і водночас якісно не змінило фізичних особливостей досліджуваного процесу. Для отримання рішень використано асимптотичні методи з малим параметром. Розглянуто три різні варіанти завдання, що відрізняються один від одного закономірностями зміни ширини каналу вздовж поздовжньої осі. В результаті рішення показано, що коли ширина каналу монотонно змінюється, також монотонно змінюється і відношення градієнтів тисків. Коли на кінцях аналізованого відрізка ширина каналу однакова, то відношення градієнтів тиску на кінцях каналу також рівні. При синусоїдальній зміні ширини каналу зміни градієнтів тиску також мають синусоїдальний характер. Основним результатом роботи є висновок про те, що при зміні ширини плоского капіляра змінюється фазове зміщення коливання тиску щодо самого себе, при цьому також змінюється і зміщення коливання витрати щодо тиску, однак сума цих зміщень залишається постійною величиною. У відомих рішеннях для постійного діаметра каналу гармонійні коливання витрати та тиску також мають фазове зміщення щодо один одного, яке залежить від діаметра каналу, але воно не змінюється вздовж каналу. Також показано, що при подальших наближеннях у рішенні з'являються компоненти, що містять гармоніки з подвоєною частотою і компонент, що не залежить від часу, тобто. при коливальному перебігу в капілярі непостійної ширини, у певних зонах утворюється незалежна від часу течія. Структура такої внутрішньої течії, як випливає з рішення, залежить від зміни діаметра капіляра і від коливання, що задається.

Ключові слова: капіляр, рідина, дифузія, масообмін, коливання.